Final exam: **Electrodynamics of Radiation Processes,**
31st January 2018, 14-17
The exam consists of 100 points in total.

Write your name and student ID number on every page.
Make certain to clearly label which answer is which on your exam papers.

Only calculators can be used. **No laptops, tablets, iPads, or other internet devices are allowed. Also no books or notes are allowed.**

Explain clearly all of the steps that you use to derive all your results. If you are not sure of a particular step make an estimate, be clear you doing this, and continue.

Make certain that your handwriting is readable to someone besides yourself.

1. **Blackbody Radiation [25 pts]**

   (a) **[5 pts]** Explain what is blackbody radiation, and what are the required properties of an astrophysical source?

   (b) **[4 pts]** Why is our Sun an almost perfect blackbody?

   (c) **[4 pts]** Given the Planck law:

   $$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

   derive the Rayleigh-Jeans law ($h\nu \ll kT$) and the Wien law ($h\nu \gg kT$)

   (d) **[6 pts]** Show that the Wien displacement law, derived from the Planck law, is:

   $$\nu_{max} = 5.88 \times 10^{10}\mathrm{T}\ \ \mathrm{Hz}$$

   [Hint: $x = 3(1 - e^{-x})$ has the approximate root, x=2.82]

   (e) **[6 pts]** The two brightest stars in the Orion constellation are Betelgeuse with a surface temperature T ~3400 K and Rigel with T ~10 100 K. Estimate the frequency at which the peak of the emission occurs for both these stars, and make a sketch of the spectrum of both on the same flux vs. frequency plot.

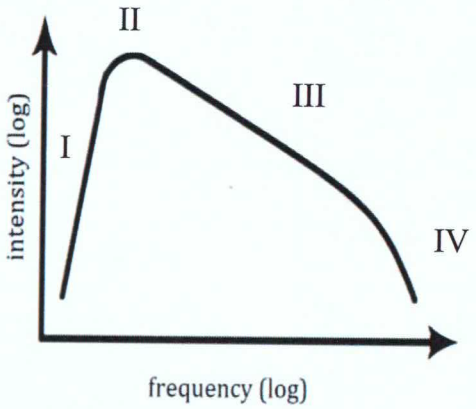

Figure 1: The synchrotron spectrum of a radio galaxy

2. **Synchrotron Radiation [25 pts]**

   (a) **[4 pts]** Under what circumstances does Synchrotron radiation occur? Give an example of an astrophysical source of Synchrotron radiation.

   (b) **[4 pts]** Explain the cyclotron frequency using the Lorentz force law:

$$F_{Lorentz} = e \cdot [\mathbf{E} + \frac{1}{c}(\mathbf{v} \times \mathbf{B})]$$

   (c) **[2 pts]** Explain how the cyclotron frequency changes into the synchrotron frequency when the velocities become relativistic.

   (d) **[6 pts]** Explain the effect of beaming for dipole radiation, include a sketch, and explain what are the implications for synchrotron radiation.

   (e) **[3 pts]** Why will the energy of the synchrotron radiation decrease with time? Describe how you can determine this rate.

   (f) **[6 pts]** Explain the different features (I, II, III, and IV) in the sychrotron spectrum of a radio galaxy shown in Figure 1.

3. **Bremsstrahlung [20 pts]**

(a) **[4 pts]** Explain what is Bremsstrahlung (also called free-free) radiation, how does it occur. Give an example of an astrophysical source of Bremsstrahlung radiation.

(b) **[2 pts]** Explain the terms in the Bremsstrahlung emissivity equation:

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

(c) **[4 pts]** Explain what is the impact parameter, b, in Bremsstrahlung. Use a sketch in your explanation. What determines the lower limit to this parameter?

(d) **[2 pts]** How are the properties of the impact parameter included in the Bremsstrahlung emissivity equation (given above)?

(e) **[4 pts]** Make a sketch in $\log(I_\nu)$ vs. $\log(\nu)$ for two different optically thin Bremsstrahlung sources of different temperatures (but the same density). Explain why the cut-offs occur where they do.

(f) **[4 pts]** Using the X-ray Bremsstrahlung spectrum of the central region of the Virgo cluster of galaxies, shown in Figure 2, estimate the temperature of the emitting gas.
[Note: The Boltzmann constant, $k = 8.6 \times 10^{-5}$ eV K^{-1}.]
What else does this measurement allow us to infer about the emitting gas?

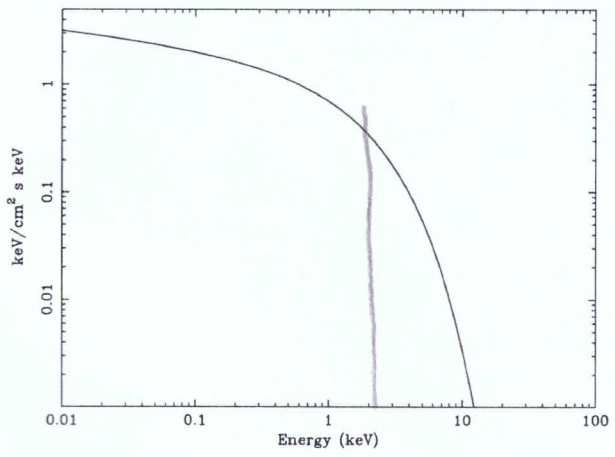

Figure 2: An X-ray spectrum of the central region of the Virgo cluster.

4. **Compton Scattering [20 pts]**

   (a) **[2 pts]** Explain what is Compton scattering and what is inverse Compton scattering.

   (b) **[5 pts]** Explain, be as quantitative as you can, how much energy can be transferred in both Compton and inverse Compton scattering.

   (c) **[4 pts]** What is the Compton y-parameter, and how is it defined? What does $y \gg 1$ mean?

   (d) **[3 pts]** Give a general (short) description of Kompaneet's Equation. What sort of equation is it? and what does it describe?

   (e) **[6 pts]** Give a general explanation of the Sunyaev-Zeldovich effect, and how it can be measured.

5. **Astrophysical source 3C 273 [10 pts]**

   Usually a variety of different radiation processes can be seen in the spectrum of an astrophysical source. Figure 3 shows the spectrum of 3C 273, which is a well studied and highly luminous nearby quasar.

   (a) **[5 pts]** Which 3 main radiation processes are clearly visible in the spectrum of 3C 273 shown in Figure 3? and what is their origin in the system? Which 4th process is assumed to make a negligible contribution?

   (b) **[2 pts]** What is the wavelength range shown in Figure 3?
   [hint: hc = 1240 eV nm]

   (c) **[3 pts]** What is Faraday rotation? and under what astrophysical circumstances can it be detected and used to study the properties of the interstellar medium?

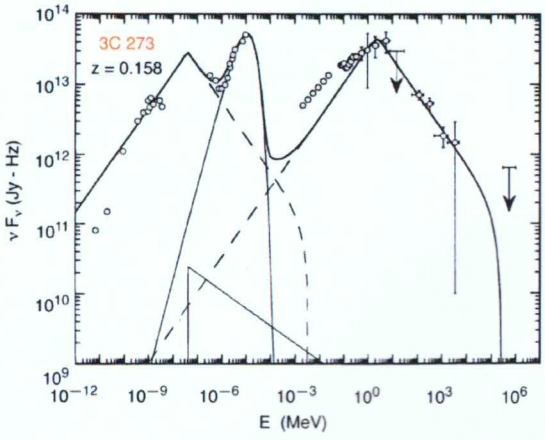

Figure 3: The spectrum of 3C 273.